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Poincaré inequalities for differential forms in Heisenberg groups: the case of continuous primitives

Abstract. In this talk I will present a group of results obtained in collaboration with Bruno Franchi (Bologna) and Pierre Pansu (Orsay). In the Euclidean setting $\mathbb{R}^N$, if $1 \leq p < N$ and $q \geq p$, we say that a (p, q) Poincaré type inequality holds for differential forms if, given a closed differential h-form ω in $L^p(\mathbb{R}^N)$, there exists a $(h-1)$-form ϕ in $L^q(\mathbb{R}^N)$, such that $d\phi = \omega$ and

$$\|\phi\|_q \leq C \|\omega\|_p,$$

for $C = C(N, p, q, h)$. When we speak about Sobolev inequalities, we have in mind compactly supported differential forms. A local version of these inequalities can be also considered (i.e. on balls). In the last few years, in collaboration with Bruno Franchi and Pierre Pansu, we have proved some global and interior (p, q) Poincaré and Sobolev inequalities for the so-called Rumin complex in Heisenberg groups. The first bunch of results have been obtained for $1 \leq p < Q$ (or $p < Q/2$, depending on the degree of the forms) where Q is the homogeneous group dimension. Also the limiting case $q = \infty$ has been considered. The endpoint situation $q = \infty$ corresponds to the exponent $p = Q$ (or $p = Q/2$). Notice that already in the Euclidean setting, when $p = N$ and $h = 1$, a Poincaré inequality fails to hold (Trudinger inequality), but Burgain & Brezis proved that Poincaré inequality on the N-torus still holds when $h = N$, $p = N$ and $q = \infty$. Even more, the potential ϕ can be taken continuous. The global version on $\mathbb{R}^N$ itself is even slightly stronger: the primitive can be taken to be continuous and to tend to zero at infinity, as proved by Moonens and Picon. In this talk we show that the result of Moonens and Picon in $\mathbb{R}^N$ holds for closed differential forms of any degree $h > 1$. In addition, we prove analogous results for the Rumin's complex in Heisenberg groups. In particular, that it possible to upgrade bounded primitives to bounded and continuous primitives vanishing at infinity.