HOMOTOPY THEORY, INFINITY CATEGORIES, APPLICATIONS: SEMINAR OUTLINE

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1. Introduction

In this seminar, we will survey recent developments in the fields of homotopy theory and higher category theory. The aim of the seminar is an updated understanding of algebraic K-theory, Hochschild homology and generalised cohomology theories. Our main references for the seminar will be [1, 2].

2. Outline of the seminar

A tentative list of topics that we plan to cover are as follows:

(1) **Simplicial homotopy theory:** simplicial objects in a category, simplicial homotopies, geometric realisation of a (bi)simplicial set, Dold-Kan correspondence;
(2) **Bar-cobar construction:** with applications such as Hochschild (co)homology and looping-delooping;
(3) **The quasi-categorical approach to ∞-categories:** Kan conditions, Joyal-Lurie definition of ∞-categories, nerve and classifying space of a category, Bousfeld-Kan homotopy colimits and limits, limits and colimits in quasi-categories, symmetric monoidal structures on ∞-categories;
(4) **Operads:** definition and examples of operads, algebras and monads associated to operads, A_∞ and E_∞-operads;
(5) **Diagram categories and infinite loop spaces:** Γ-spaces and Γ-objects in a category, spectra associated to a Γ-spaces (for example, the sphere spectrum and the Eilenberg-Maclane spectrum), smash product of Γ-spaces, the θ_n-category in relation to n-loop spaces, and as a model for (∞, n)-categories;
(6) **Presentation of an ∞-category:** model categories, combinatorial model categories and presentable ∞-categories;
(7) **Spectra and its universal property:** symmetric monoidal ∞-categories via cocartesian fibrations, stabilisation and stable ∞-categories, the stable, presentable ∞-category of spectra, smash product of spectrum objects.
(8) **Functor homology (time-permitting):** definition and application towards axiomatic description of Tor and Ext.

Referencias

[1] Birgit Richter, *From categories to homotopy theory* **188** (2020).
[2] Moritz Groth, *A short course on infinity categories*, arXiv preprint arXiv:1007.2925 (2010).

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