Title. The equations of the Newtonian n-body problem: some subtle properties.

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Abstract. The traditional teaching of classical mechanics presents the Newtonian mechanics, then the Lagrangian mechanics, then the Hamiltonian mechanics. The first is often considered as a dusty chapter which does not deserve the attention of a mathematician. Nothing is done to give a precise meaning to, e.g., the so-called law of action and reaction. An accumulation of undefined words from the past centuries gives the impression of a piece of science which should be kept in a museum. But the Newtonian mechanics is still in many physical contexts the only theory which describes the phenomena. We will present the equations of multiparticle interactions, as for example the equations of the n-body problem (with masses m_i and positions q_i),

$$\frac{d^2 q_i}{dt^2} = \sum_{j \neq i} \frac{m_j}{r_{ij}^3}(q_j - q_i), \qquad i = 1, \ldots, n, \qquad r_{ij} = \|q_i - q_j\|,$$

as the mathematical objects which can replace the old undefined words. Their rich structure will be presented in modern words, related to Galilean symmetries and conserved quantities, even in dissipative contexts. Some attention will be given to the Lagrange equilateral solutions of the 3-body problem and their generalizations. They will be related to the subtle symmetry properties of the equations.

Some references: see in my preprint page: perso.imcce.fr/alain-albouy/albo_preprint.html

On a paper of Moeckel on Central Configurations, Mutual distances in Celestial Mechanics